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Identification and estimation of thresholds in the fixed effects ordered logit model

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ABSTRACT

This paper proposes a new estimator for the fixed effects ordered logit model. In contrast to existing methods, the new procedure allows estimating the thresholds. The empirical relevance and simplicity of implementation is illustrated in an application on the effect of unemployment on life satisfaction.

1. Introduction

There exist a number of proposals in the literature on how to estimate a panel-ordered logit model with individual fixed effects—Das and van Soest (1999), Ferrer-i-Carbonell and Frijters (2004) and most recently Baetschmann et al. (2011). A drawback of all these estimators is that they do not identify the threshold parameters. This paper proposes a new estimating procedure which allows estimating these thresholds. Knowing the thresholds has three advantages: first, the thresholds are helpful for interpreting the regression coefficients; second, they make it possible to obtain statements about the effect of a changing x on the observed ordered variable and not only on the latent variable; and third, comparing the differences between the thresholds can be interesting in itself. The new procedure can be easily implemented using existing software for conditional maximum likelihood (CML) logit estimation with cluster corrected standard errors.

The paper proceeds as follows. Section 2 presents the fixed effects ordered logit model and discusses the new estimation procedure. In Section 3 the new estimator is applied to data from the German Socioeconomic Panel.

2. Econometric methods

2.1. The FE ordered logit model

The fixed effects ordered logit model relates the latent variable y^*_{it} for individual i at time t to a linear index of observable characteristics x_{it} and unobservable characteristics α_i and ε_{it}:

$$y^*_{it} = x'_{it}\beta + \alpha_i + \varepsilon_{it}, \quad i = 1, \dots, N \; t = 1, \dots, T. \tag{1}$$

The time-invariant part of the unobservables (α_i), called the fixed effect, can be statistically dependent on x_{it}.

The latent variable is tied to the (observed) ordered variable y_{it} by the observation rule:

$$y_{it} = k \quad \text{if } \tau_k < y^*_{it} \le \tau_{k+1}, k = 1, \dots, K, \tag{2}$$

where the thresholds τ are assumed to be strictly increasing ($\tau_k < \tau_{k+1} \; \forall k$) and $\tau_1 = -\infty$, $\tau_{K+1} = \infty$.

The specification of the fixed effects ordered logit model is completed by assuming that the ε_{it} are conditionally independent and identically standard logistically distributed. I.e., if $F(\cdot)$ denotes the cdf

$$F(\varepsilon_{it}|x, \alpha) = \frac{\exp(\varepsilon_{it})}{1 + \exp(\varepsilon_{it})} \equiv \Lambda(\varepsilon_{it}). \tag{3}$$

Hence, the probability of observing an outcome equal to k for individual i at time t using (1)–(3) can be written as

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$$\Pr(y_{it} = k|x_{it}, \alpha_i) = \Lambda(\tau_{k+1} - x'_{it}\beta - \alpha_i) - \Lambda(\tau_k - x'_{it}\beta - \alpha_i), \quad (4)$$

whereas the probability of an outcome greater than or equal to k is

$$\Pr(y_{it} \geq k|x_{it}, \alpha_i) = \Lambda(x'_{it}\beta + \alpha_i - \tau_k). \quad (5)$$

Eqs. (4) and (5) show that the location of the τ's and α's cannot be distinguished. Thus the constant and the second threshold (τ_2) are normalized to zero.

The problem with maximum likelihood estimation based on (4) is that the expression depends on the individual fixed effect α_i. Including individual dummies in the estimation procedure to account for fixed effects is not a solution due to the "incidental parameter problem"—e.g. Chamberlain (1980).

2.2. Illustration of the estimation procedure

The binary logit model is one of the few nonlinear models, where it is known how to deal with fixed effects. For this model, Chamberlain (1980) proposed to condition the likelihood on the number of one's in an individual's record to get rid of the individual fixed effects. Chamberlain's method can be applied to the ordered logit model as well. The procedure is as follows: first, the ordered dependent variable y is dichotomized to a binary one. The binary variable is denoted by d and the cutoff by k: $d = \mathbb{1}(y \geq k)$, where $\mathbb{1}()$ is the indicator function. Second, Chamberlain's estimation procedure is applied to d.

To illustrate this procedure consider an individual which is observed two times, where the first observation equals 3 and the second equals 1: $y_1 = 3, y_2 = 1$. We assume that the variable y can take the values 1, 2 and 3, thus k can be either 2 or 3. In this example both choices result in the same binary dependent variable, $d_1 = 1$ and $d_2 = 0$. We obtain the conditional probability in Box I.

The last expression in (6) is independent of α. Thus β can be estimated by the conditional maximum likelihood.

The problem with this procedure is that the τ's disappear from the probability expression as well and are therefore not identified. The reason is that the same cutoff is used for all observations of an individual, so there is no "cutoff variation" within a conditional likelihood contribution. By contrast, if the observations of an individual are dichotomized at different cutoff points and the probability expression is applied accordingly, the thresholds are identified. Consider again the above example, but suppose now that the first observation is dichotomized at 2 and the second at 3. The probability that the first dichotomization is 1, given that either the first or the second (but not both) is now given in Box II.

This expression is independent of α but depends on β and the τ's. Hence there is no incidental parameter problem, and β and τ can be estimated by maximum likelihood. The method can easily be generalized to situations with more than two time periods and more than two possible cutoff points.

2.3. Practical implementation—choosing the cutoff points

The question arises: which combinations of observation specific cutoff points to include in the estimation procedure? One possibility is to include all feasible combinations. Cluster standard errors can be used to account for the dependence between the conditional likelihood contributions of the same individual (White, 1982). The same idea of including more than one "clone" of an individual combined with cluster standard errors is used by the BUC estimator (Baetschmann et al., 2011) to estimate β in the FE ordered logit model with individual specific thresholds. In the previous example, there are four combinations: both observations can be dichotomized at two cutoff points and all four combinations are possible. Among those, only the two combinations with different cutoff points are informative for estimating the τ's.

Table 1
Fixed effects ordered logit estimates of life satisfaction.

Dep. var. life satisfaction	$y \geq 8$		BUC_τ	
Unemployed	-0.96^{**}	(0.22)	-1.10^{**}	(0.18)
Out of labor force	-0.24	(0.13)	-0.48^{**}	(0.12)
Duration of unemployment	-0.01	(0.02)	-0.02	(0.02)
Squared duration of unemp. $\times 10^{-4}$	0.60	(3.54)	2.45	(2.54)
Married	0.67^{**}	(0.14)	0.59^{**}	(0.12)
Good health	0.34^{**}	(0.06)	0.35^{**}	(0.05)
Age	-0.12^{**}	(0.04)	-0.11^{**}	(0.03)
Squared age $\times 10^{-2}$	-0.84	(4.56)	-1.07	(3.95)
Log. household income	0.13^{*}	(0.06)	0.13^{*}	(0.05)
τ_3			0.69	(0.15)
τ_4			1.35	(0.15)
τ_5			2.17	(0.15)
τ_6			2.90	(0.15)
τ_7			4.31	(0.16)
τ_8			5.06	(0.16)
τ_9			6.28	(0.16)
τ_{10}			7.97	(0.17)
τ_{11}			9.17	(0.17)
Observations	12,980		204,574	
Individuals	2573		4204	

Notes: Data source GSOEP, waves 1984–1989.
* Statistical significance at 5% level.
** Statistical significance at 1% level.
Cluster robust standard errors in parentheses.

The inclusion of all possible cutoff point combinations in the estimation procedure is only feasible if the number of time periods (T) and the number of categories (K) of the dependent variable are small, because the number of possible combinations is $(K-1)^T$. For example if T and K are equal to 10, there exist more than three billion possible copies of each individual. Often the researcher is more interested in estimating β than τ. On this account, I propose to include all clones with no variation in the cutoff to estimate β precisely and fill up the rest of the dataset with a limited number of clones with random variation in the cutoff points. (Stata code is available from the author upon request.)

3. Illustration

To illustrate the estimation procedure, the new fixed effects ordered logit estimator is applied to the model and dataset of Winkelmann and Winkelmann (1998). The dataset consists of a sample from the German Socioeconomic Panel going from 1984 to 1989 with 4261 individuals. The dependent variable is satisfaction with life, which is measured as the answer to the question: "How satisfied are you at present with your life as a whole?". The answers ranges from 0, "completely dissatisfied", to 10, "completely satisfied". To be consistent with the notation of the theoretical part of this paper, the dependent variable is recoded and ranges now from 1 to 11.

If each individual would be dichotomized in all possible ways, the resulting dataset would consist of more than four billion entries. Hence I decided to include all clones with a constant cutoff to estimate β precisely, plus ten clones of each individual, whose observations are dichotomized at observation specific random cutoff points. Compared to other proposed estimators, individuals without variation in the ordered dependent variable are not automatically excluded from the estimation procedure. The reason is that variation in the ordered dependent variable (y) is not a precondition for variation in the dichotomized dependent variable (d) if varying cutoffs within a conditional likelihood contribution are allowed.

The columns with the heading "$y \geq 8$" in Table 1 show the estimates of the Chamberlain "estimator" if the ordered variable is dichotomized at 8. These estimates are also reported in Winkelmann and Winkelmann (1998). The results of the new estimation procedure are very similar and are listed in the columns with the heading "BUC_τ". The standard errors of the new estimator

$$\begin{aligned}
&\Pr[d_1 = 1 \cap d_2 = 0 | (d_1 = 1 \cap d_2 = 0) \cup (d_1 = 0 \cap d_2 = 1)] \\
&\quad = \Pr[y_1 \geq k \cap y_2 < k | (y_1 \geq k \cap y_2 < k) \cup (y_1 < k \cap y_2 \geq k)] \\
&\quad = \frac{\frac{\exp(x_1'\beta+\alpha-\tau_k)}{1+\exp(x_1'\beta+\alpha-\tau_k)} \frac{1}{1+\exp(x_2'\beta+\alpha-\tau_k)}}{\frac{\exp(x_1'\beta+\alpha-\tau_k)}{1+\exp(x_1'\beta+\alpha-\tau_k)} \frac{1}{1+\exp(x_2'\beta+\alpha-\tau_k)} + \frac{1}{1+\exp(x_1'\beta+\alpha-\tau_k)} \frac{\exp(x_2'\beta+\alpha-\tau_k)}{1+\exp(x_2'\beta+\alpha-\tau_k)}} \\
&\quad = \frac{\exp(x_1'\beta + \alpha - \tau_k)}{\exp(x_1'\beta + \alpha - \tau_k) + \exp(x_2'\beta + \alpha - \tau_k)} = \frac{\exp(x_1'\beta)}{\exp(x_1'\beta) + \exp(x_2'\beta)}
\end{aligned} \tag{6}$$

Box I.

$$\begin{aligned}
&\Pr[y_1 \geq 2 \cap y_2 < 3 | (y_1 \geq 2 \cap y_2 < 3) \cup (y_1 < 2 \cap y_2 \geq 3)] \\
&\quad = \frac{\frac{\exp(x_1'\beta+\alpha-\tau_2)}{1+\exp(x_1'\beta+\alpha-\tau_2)} \frac{1}{1+\exp(x_2'\beta+\alpha-\tau_3)}}{\frac{\exp(x_1'\beta+\alpha-\tau_2)}{1+\exp(x_1'\beta+\alpha-\tau_2)} \frac{1}{1+\exp(x_2'\beta+\alpha-\tau_3)} + \frac{1}{1+\exp(x_1'\beta+\alpha-\tau_2)} \frac{\exp(x_2'\beta+\alpha-\tau_3)}{1+\exp(x_2'\beta+\alpha-\tau_3)}} \\
&\quad = \frac{\exp(x_1'\beta + \alpha - \tau_2)}{\exp(x_1'\beta + \alpha - \tau_2) + \exp(x_2'\beta + \alpha - \tau_3)} = \frac{\exp(x_1'\beta - \tau_2)}{\exp(x_1'\beta - \tau_2) + \exp(x_2'\beta - \tau_3)}
\end{aligned} \tag{7}$$

Box II.

are slightly smaller. The essential advantage of the new estimation procedure is that estimates for the thresholds are available. The differences between them ranges from 0.66 between τ_3 and τ_4 to 1.69 between τ_9 and τ_{10}. Roughly speaking, the differences between the thresholds increase with the threshold number. This means that the effect of an increasing latent index—for example, by $+1.10$ when being employed rather than unemployed—on the ordered life satisfaction variable is largest for people with a low life satisfaction level.

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